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Abstract

Frequency hopping is a feature in GSM (Global System for Mobile Communications) cellular systems in which a frequency carrying the communication rapidly changes over time. This improves the overall capacity or the communication quality without the addition of new resources. The frequency hopping problem involves the determination of the subset of frequencies allocated to each cell to be used for hopping, as well as determining the hopping sequence of each radio in the network. This paper investigates this latter aspect for three levels of network synchronization. Heuristic procedures for evaluating and generating hopping sequences are also proposed. Results from a real-life network indicate that by exploiting the control offered by higher synchronization levels, it is possible to significantly improve interference levels and to further take advantage from the gains offered by frequency hopping.

Keywords: Cellular networks, frequency hopping, heuristic optimization.

Résumé

Le saut de fréquence est une caractéristique des réseaux GSM qui permet une meilleure exploitation de la bande radio. En changeant la fréquence porteuse des communications rapidement, le saut de fréquence génère ainsi une augmentation de la capacité du réseau ou une amélioration de la qualité du service. Le problème de gestion de réseaux GSM avec saut de fréquence implique l'allocation de sous-ensembles de fréquences aux cellules du réseau et la détermination des séquences de saut pour chacune des radios dans le réseau de telle sorte que la demande en fréquences soit satisfaite, et que l'interférence soit minimisée. Cet article analyse ce deuxième volet du problème dans le contexte de trois niveaux de synchronisation du réseau. Des heuristiques pour l'évaluation et la génération de séquence de saut de fréquence sont aussi proposées. Les résultats, obtenus pour un réseau réel, indiquent que l'introduction de la synchronisation dans le réseau permet une meilleure exploitation des avantages offerts par le saut de fréquences.

Mots clés : réseaux cellulaires, saut de fréquence, optimisation heuristique.

1. Introduction

Frequency Hopping (FH) is a feature in GSM-type networks that aims at overcoming high interference levels through the sharing of interference among users in the network. The FH concept was first introduced in 1940 by Hedy Lamarr and George Antheil (IEEE History Center (2003)). FH involves broadcasting a signal over a changing series of radio frequencies, quickly switching from frequency to frequency. The initial objective was to reduce the danger of detection for military radio communications. In modern cellular communication systems, FH also allows a better exploitation of the available radio spectrum.

In this research, we analyze and optimize the radio spectrum utilization under FH, using a two-step approach: 1) allocating frequencies to cells and 2) determining the frequency hopping sequences for each radio in the network. In Bourjolly et al. (2002), the question of allocating frequencies to cells in an FH-network was examined, with a proposal for a new type of optimized hopping pattern in synchronized networks. In this paper, we consider the problem of determining hopping sequences according to the hopping schemes that are currently implemented within the GSM standards. For this, we distinguish between three levels of network synchronization. For each level, we propose a heuristic procedure to optimize the GSM parameters that control the hopping sequences.

In the remainder of this section we describe frequency hopping in GSM networks in more detail. Section 2 describes our approach to evaluating frequency hopping plans. Section 3 provides the model used to manage frequency hopping and describes the proposed heuristic procedure to generate hopping plans. Finally in Section 4, the proposed approach is tested on a real-life network.

A GSM network consists of a set of *antennas* scattered around the service area, each of which provides service to a relatively small area called a *cell*. A group of radio *transceivers* (TRXs) are connected to each antenna. When a *mobile unit* (the user) makes or receives a call, the communication is established through the TRXs using radio frequencies. Each TRX accommodates eight communication channels, through time sharing known as TDMA (*Time Division Multiple Access*). The first time slot on the first

TRX of a cell is used as the *Broadcast Control Channel* (BCCH). The remaining time slots in that TRX and the time slots in the other TRXs are used as *Traffic Channels* (TCH) to carry communications. Because the BCCH is used for establishing calls and for signaling purposes, the TRX carrying it is not allowed to hop.

The *cell allocation* (CA) is the subset of frequencies allocated to a cell. Under static frequency allocation, the CA contains a number of frequencies equal to the number of TRXs (one frequency for each TRX). The *mobile allocation* (MA) is the subset of frequencies from the CA that is allocated to a particular call. Under FH, the BCCH-carrying TRX has an MA containing only one frequency (since no hopping may occur). We consider *Synthesized* FH, which can be supported by current equipment. With this type of hopping, TRXs are capable of retuning to different radio frequencies. Older equipment can support only *Baseband* FH since they can be tuned to one frequency only. This limits the size of the CA to the number of TRXs of the cell, while under Synthesized FH, the limit is set by the GSM standard at 64 frequencies.

GSM exploits the ability of the radio units (the transceivers and mobile units) to transmit and receive on different frequencies, in order to implement slow Frequency Hopping. FH is a technique where the frequency used for the communication between a transceiver and a mobile unit is allowed to change over time at a prescribed rate (217 times per second). Detailed description of the hopping mechanism and sequence generation procedure can be found in ETSI (1999). Both the transmitter and the receiver need to have the same hopping sequence so that they are always tuned to the same frequency. In GSM, two types of frequency hopping patterns are used:

- under *Random Hopping* (RH), the hopping sequence is randomly generated according to a deterministic pseudorandom generator;
- under *Cyclic Hopping* (CH), the hopping sequence cycles through the available frequencies in the MA from the lowest index to the highest index.

In GSM, the *Hopping Sequence Number* (HSN) is the parameter that determines the type of hopping pattern to be used: if HSN is set equal to 0 then the Cyclic Hopping is active; if HSN is set to an integer value in the interval [1,63], then random hopping is

active and each value corresponds to a different hopping sequence. Each cell is allocated one HSN value.

The sequence generator in GSM also depends on the *Frame Number* (FN) which is a time counter. Each TDMA frame (of length 8 x 3/5200 seconds) is numbered by the frame number. It cycles in the range [0,H] (where H = 26 x 51 x 2048 = 2715647). By design, calls connected to the same cell are aligned, i.e., they have the same FN.

The *Mobile Allocation Index Offset* (MAIO) is another parameter involved in the sequence generator. It is used to avoid that two aligned TRXs with the same HSN (i.e., the same hopping sequence) interfere with each other. The MAIO makes sure that the two TRXs start hopping from different positions in the hopping sequence. For instance, consider the case of TRXs belonging to the same cell. Since these TRXs are part of the same Base-Station, the hopping sequences are aligned and also the interference level would be unacceptably high if they used the same hopping pattern (they would be using the same frequencies all the time). To avoid this situation, the hopping sequences for these TRXs are made orthogonal by adjusting them to different MAIOs.

For a given TRX with specified MA, HSN, MAIO and at a given FN, the *Mobile Allocation Index* (MAI) indicates the index (from 0 to |MA|-1) of the frequency in the MA to be used during frame FN. The MAI is the output of the hopping sequence generator.

The frequency hopping problem in GSM networks can be now be stated as consisting of:

- determining the MA of each TRX in the network,
- assigning an HSN to each cell,
- assigning an MAIO to each TRX.

The aim behind the introduction of FH is the reduction of interference. From a signal processing point of view, frequency hopping has two types of direct effects: interference diversity and frequency diversity (Ivanov et al. (1996), Nielsen and Wigard (2000)):

- Interference diversity results from the change in the set of TRXs using the same frequency over time. Any TRX would experience high interference level for a short amount of time only. Continuous interference can cause several

bursts in a row to be erroneous. The signal coding in GSM would perform poorly in such cases. FH reduces the probability of consecutive corrupted bursts so that the coding mechanism can work better.

- The signal propagation occurs on multiple paths resulting in variations in the power of the received signals, and hence in a strong fading of the signal. This negatively impacts the signal quality. With frequency diversity, this effect is reduced since only some isolated bursts will be affected (which can be corrected with the error correcting code incorporated in GSM communication).

Dynamic frequency hopping (DFH) has been proposed to improve upon the conventional frequency hopping (Kostic et al. (2000)). The idea here is to use dynamic resource allocation for the purpose of interference avoidance: while in conventional FH the hopping patterns are pre-determined (given the MAs, HSNs and MAIOs assignments), with DFH, the hopping patterns are built in real-time in order to adapt to changes in the interference, which in turn depends on the set of active users. Under static frequency allocation and conventional FH, a worst-case scenario is often considered, where all TRXs are assumed to be active. Hence, DFH is able to cope with changes in the distribution of mobile units in time and space. However, DFH requires continuous measurement of the interference and the transmission of these measurements to some central controller. This implies an overhead cost on the network that may be significant. A second requirement for DFH is system wide synchronization. As indicated in Kostic et al. (2000), such a feature would be feasible in wireless standards such as EGPRS (an extension of GSM) and IS-136.

Optimized hopping (Bourjolly et al. (2002)) is another alternative that has been proposed to improve upon conventional FH for synchronized networks. As for DFH, this approach also does not use the sequence generator currently implemented in GSM, but rather proposes generating the sequences offline, hence avoiding the overhead costs associated with online management.

When it comes to using the hopping sequence generator currently implemented in GSM networks, the HSN-MAIO assignment problem has not received much attention.

The HSN and the MAIO parameters are used to control FH in GSM networks by creating virtually uncorrelated hopping sequences. Gamst (1991) examined the problem of managing the HSN. The problem is modeled as a graph coloring problem on a graph whose nodes correspond to cells and whose edges represent the existence of a common frequency in the MAs of the corresponding cells. The resulting HSN assignment problem prohibits the use of the same HSN for a pair of cells if they use at least one common frequency. Nielsen and Wigard (2000) recommend a similar approach to managing the HSN allocations. The underlying assumption of this approach is that sequences with different HSN are not correlated. This is contradicted by the findings of Nyberg et al. (2000).

In fact, Nyberg et. al. (2000) study the collision properties of random hopping sequences as a function of the MA, HSN, MAIO and FN. Here, the collision rate between two sequences is defined as the fraction of bursts for which the two sequences use interfering frequencies. For independent random sequences the collision rate should depend on the MA allocation only. The authors show that these sequences are in reality correlated, and the level of correlation depends on the assigned HSN-MAIO values in addition to the MA allocations. This implies that the HSN, MAIO and FN values jointly determine the real level of collisions between any two sequences. Therefore, just assigning different values for the HSN or MAIO may not guarantee that the resulting sequences will have a low collision rate.

Nyberg et al. (2000) suggest generating a correlation table for all possible parameters values. For synchronized networks, the authors suggest using this table in an on-line mode for hopping sequence selection. The on-line requirement stems from the fact that the level of correlation depends on the difference in the FN of the two sequences. Since hops occur at the same time in synchronized networks, the use of highly correlated sequences would result in systematic interference, which is exactly what FH is designed to avoid. For asynchronous networks, the authors suggest computing the spread of collision rates as a function of the MAIO and FN and minimizing such spreads. The difficulty of this approach is that for every pair of TRXs, we need to calculate the collision rate for all possible HSN-MAIO combinations over a range of FNs. This involves significant computational and memory requirements.

While radio spectrum management in a static environment has received considerable attention, little work has been done in the context of frequency hopping. In the next section, we describe our approach to evaluating HSN-MAIO allocation plans and propose a mathematical formulation for this problem. In Section 3 we propose a solution construction heuristic.

2. Optimizing HSN-MAIO Allocation

Hopping sequences are the output of the deterministic sequence generator already implemented in GSM. In this case, hopping sequences are generated by determining an HSN-MAIO plan. For all TRXs, under Random or Cyclic hopping, any sequence s results from an HSN assignment h to its cell and an MAIO assignment m. So we denote $s=(h,m)$.

We can define the following:

- $x_{ch} = \begin{cases} 1, \text{if cell } c \text{ is assigned HSN } h \\ 0, \text{otherwise} \end{cases}$
- MA_c is the frequency allocation of hopping TRXs in cell c
- $y_{im} = \begin{cases} 1, \text{if TRX } i \text{ is assigned MAIO } m \in \{0,.., MA_c - 1\} \text{ where } i \in c \\ 0, \text{otherwise} \end{cases}$
- $C^{co}_{(is,jr)}$ is the co-channel collision rate that results from using sequence $s=(h,m)$ in TRX i and sequence $r=(h',m')$ in TRX j
- $C^{ad}_{(is,jr)}$ is the adjacent-channel collision rate that results from using sequence $s = (h,m)$ in TRX i and sequence $r = (h',m')$ in TRX j
- Co_{ij} is the co-channel interference level between TRX i and TRX j
- Ad_{ij} is the adjacent-channel interference level between TRX i and TRX j
- $T_{(is,jr)} = Co_{ij} C^{co}_{(is,jr)} + Ad_{ij} C^{ad}_{(is,jr)}$ is the total collision cost associated with using sequence $s = (h,m)$ in TRX i and sequence $r= (h',m')$ in TRX j.

One can impose constraints to prohibit the simultaneous use of a pair of sequences at a pair of TRXs if this causes a high level of interference. For instance, such

a simultaneous use of sequence s at TRX i and sequence r at TRX j is prohibited if the following condition holds:

$$Co_{ij} C^{co}_{(is,jr)} > \vartheta^{co}$$

or

$$Ad_{ij} C^{ad}_{(is,jr)} > \vartheta^{ad}, \tag{1}$$

where, ϑ^{co} and ϑ^{ad} are some prescribed threshold values. Such conditions hold if the joint effects of interference level (co-channel or adjacent channel) and collision rates are too high.

The simultaneous HSN-MAIO assignment problem can now be formulated as follows:

$$\underset{x_{ch},\, y_{im}}{\text{Minimize}} \quad \sum_{\substack{i \in c_1,\, j \in c_2 \\ s=(h,m),\, r=(h',m')}} T_{(is,jr)} (x_{c_1 h} y_{im})(x_{c_2 h'}, y_{jm'}) \tag{2}$$

Subject to:

$$\sum_{h} x_{ch} = 1 \qquad \forall\ c \tag{3}$$

$$\sum_{m} y_{im} = 1 \qquad \forall\ i \tag{4}$$

$$x_{c_1 h} y_{im} + x_{c_2 h'} y_{jm'} \le 1 \qquad \forall\ c_1, c_2, i \in c_1, j \in c_2, \text{ for which (1) holds and where } s = (h,m) \text{ and } r = (h',m') \tag{5}$$

$$x_{ch}, y_{im} \in \{0,1\} \qquad \forall\ c,i,h,m \tag{6}$$

In the above formulation, the objective function (2) attempts to minimize the total expected interference. Constraints (3) ensure that every cell is assigned one HSN value and constraints (4) ensure that every hopping TRX is assigned one MAIO value. For BCCH-carrying TRXs, we have $y_{i0} = 1$ (i.e. automatically assigned to MAIO 0 since the MA contains only one frequency). Constraints (5) forbid the joint assignment of a pair of sequences that have an excessive collision cost. Instead of using constraints (5), we can replace the objective function coefficients corresponding to the prohibited pair of sequences by a large number.

The difficulty with this formulation for the HSN-MAIO assignment problem lies in computing the $T_{(is,jr)}$ values. In order to get an idea about the number of such values that need to be computed, we denote:

- C: a set of cells,
- d_c: the number of TRXs in cell c,
- MA_c: the mobile allocation of hopping TRXs in cell c.

There are 64 possible HSN values for each cell and, for each TRX, the MAIO can have any of MA_c values. Thus, in the above model,

- the number of x_{ch} variables is $64|C|$,
- the number of y_{im} variables is $\sum_{c \in C}(|MA_c|-1)(d_c-1)$,
- for each pair of cells we need to compute the collision between all possible sequences. Thus the total number of $T_{(is,jr)}$ parameters to be computed is $(64|MA_c|)^2|C|^2/2$,
- for example: if $|C|=100$ cells, $d_c=3$ and $|MA_c|=6$ for all cells, then the model involves 8200 variables and the computation of about 738 million $T_{(is,jr)}$ terms.

Given the huge number of simulations required to compute the $T_{(is,jr)}$ terms, we need to reduce the model size. A first approach is to restrict the model by looking at the possible collisions between a small subset of cells S_c for each cell c, and hence, making the implicit assumption that collisions with other cells not in S_c are not significant. For instance, we can identify for each cell c a set S_c of critical neighbors. This set may contain all cells with high potential interference with cell c.

The above problem simplification may still require running a significant number of simulations. A second approach would be to decompose the problem: first assign an HSN value to each cell as suggested in Gamst (1991) and Nielsen et al. (2000), and in a second step, assign an MAIO value to each TRX. In Gamst (1991), the proposed model consists of assigning HSNs to cells such that no two cells whose MA share a frequency can be allocated the same HSN. Given a fixed HSN assignment, it now remains to assign the MAIOs to TRXs. This two-step HSN-MAIO allocation procedure is based on the idea

that for a given pair of cells that are sharing a frequency, the collision rates are more significant if they are assigned the same HSN. This is not necessarily true. In Nyberg et al. (2000), it was shown that the collision depends on HSN, MAIO and FN. Because of this, we opt to assign both HSN and MAIO simultaneously.

So far we have not addressed the issue of calculating the $T_{(a,b)}$ parameters. This depends on network synchronization. As illustrated in Figure 1, we distinguish between three types of networks. As the name implies, asynchronous networks are those with no synchronization at all. Thus if one considers any two TRXs belonging to different cells, their corresponding frame numbers can take any value in the allowed range (*0* to *H*). We call Hop-synchronized networks those where the frequency switch occurs at the same time, but the frame number for any TRX pair could be different (ΔFN is the difference in frame number between two TRXs). The third type, fully-synchronized networks, corresponds to the case where the frame number is always the same for all TRXs at any time. This distinction is useful for evaluating hopping sequences.

Inset figure 1 here

The next paragraphs describe the procedures for calculating these parameters and the HSN-MAIO assignment procedures corresponding to each level of synchronization. The input to the procedures are the list of cells and the number of associated TRXs, the cell CA list as well as the co-channel and adjacent channel interference matrices.

2.1. HSN-MAIO assignment in a Fully-Synchronized network

For a fully-synchronized network one can evaluate the co-channel and adjacent channel collision rates between hopping sequences given an MA, HSN and MAIO assignment for each TRX. This is done by generating all sequences and counting all instances of co-channel or adjacent channel hits. This is possible since all TRXs share the same frame number (FN).

In GSM, the complete hopping sequence has a length of *H* (where H = 26x51x2048). Any algorithm designed to optimize the HSN-MAIO assignment requires

the generation of several sequences for each TRX (corresponding to different HSN and MAIO choices) and computing collision rates between pairs of sequences. Thus, generating the complete sequences is very time consuming and hence it is preferable to generate just a partial sequence. In addition collision rates computed over a partial range are not likely to differ significantly from those computed over the full sequence length as long as the partial range is not too short. In our experimental investigation we use a length of one Superframe (defined in GSM as 26x51 TDMA frames).

For the HSN-MAIO assignment in a fully-synchronized network, we use a heuristic procedure. Starting from an empty solution, the heuristic sequentially builds a partial solution by considering each cell in turn. For each cell, the heuristic determines the best cell HSN and the best combination of MAIO assignments for all of its TRXs. The determination is such that it causes the minimum increase in the objective function (2) in the HSN-MAIO model described above. We denote by $s_i^* = (h_c^*, m_i^*)$ the assignment of HSN h_c^* to cell c and MAIO m_i^* to TRX i of cell c. The assignment procedure can be summarized as follows:

Optimized HSN-MAIO assignment: Case of Fully Synchronized networks

1. Initialization
   a. Let $D = \varnothing$ and $D^0 = C$.
   b. Randomly order the cells in D□.
2. Considering each cell $c \in D^0$ in order:
   A. For all possible HSN assignments, h_c
      i. For all possible MAIO combinations $\{m_i, \forall i \in c\}$:

         Compute all $T_{(is_i, jr_j^*)}$ where j is a TRX that belongs to cells c'∈ D and

         where $s_i = (h_c, m_i)$ and $r_j^* = (h_{c'}^*, m_j^*)$.

      ii. Let $\Delta T^* = \min_{(h_c, m_i)} \sum_{i \in c} \sum_{j \in c' \in D} T_{(is_i, jr_j^*)}$ and

         $(h_c^*, m_i^*) = \arg\min_{(h_c, m_i)} \sum_{i \in c} \sum_{j \in c' \in D} T_{(is_i, jr_j^*)}$.

iii. Let $D := D \cup \{c\}$ and $D^0 := D^0 \setminus \{c\}$.

The above procedure builds a solution for the HSN-MAIO assignment problem in a fully-synchronized network. It could be further improved by running the procedure a number of times and starting each time with a differing ordering of the cells.

2.2. HSN-MAIO assignment in a Hop-Synchronized network

In a hop-synchronized network, the hopping takes place at the same time but TRXs in different cells do not have the same FN. Thus at a particular TDMA frame, if TRX *i* is using frequency *f*, then the frequency *g* being used at another TRX *j* depends on the difference in *FN* between the cells (for a fixed MA, HSN and MAIO assignment). In addition, for a hop-synchronized network, if at any instance, two TRXs use interfering frequencies, then we know that the duration of this interference lasts for exactly one frame. This is to be opposed to the asynchronous case where this does not hold.

In a hop-synchronized network, one needs to know the FN for each TRX in addition to knowing the MA, HSN and MAIO assignment in order to compute the collision rates between any two TRXs. However, the difference in FN of two TRXs (ΔFN) is not known in advance. Therefore, we propose that the HSN-MAIO planning be done by averaging the collision rates between TRX pairs over all possible ΔFN values. Define :

- ΔFN, the difference in frame number between TRXs *i* and *j*. Note that if *i* and *j* belong to the same cell then $\Delta FN=0$ since TRXs in the same cell are aligned. Since FN can take any value in the range 0 to *H*, then ΔFN can take values in the range 0 to *H-1*.
- $C^{co}_{(is,jr,\Delta FN)}$ and $C^{ad}_{(is,jr,\Delta FN)}$ are the co-channel and adjacent channel collision rates resulting from using sequence $s=(h,m)$ in TRX *i* and sequence $r=(h',m')$ in TRX *j* with a frame number difference of ΔFN.

Averaging over all possible ΔFN, we obtain:

$$C^{co}_{(is,jr)} = \frac{\sum_{\Delta FN}^{H-1} C^{co}_{(is,jr,\Delta FN)}}{H},$$

$$\text{and } C^{ad}_{(is,jr)} = \frac{\sum_{\Delta FN}^{H-1} C^{ad}_{(is,jr,\Delta FN)}}{H}.$$

Similar to the fully-synchronized case, computing $C^{co}_{(is,jr,\Delta FN)}$ and $C^{ad}_{(is,jr,\Delta FN)}$ over all the sequence length (H) is very time consuming. In the hop-synchronized network these computations need to be performed an even larger number of times (because of the averaging over possible values of ΔFN). Thus for the same reasons as for the fully-synchronized network, we also generate the sequences over a partial range. In our experimental investigation we use a length of one Superframe (defined in GSM as 26x51 TDMA frames).

The computational times are still too high as the collision rates need to be computed over the full range of possible ΔFN. To overcome this difficulty, we compute $C^{co}_{(is,jr)}$ and $C^{ad}_{(is,jr)}$ by sampling over the possible values of ΔFN. For our experimental investigation this sample has a size of 100 points. Initial experimentation indicated that without this sampling, just evaluating a given HSN-MAIO plan takes over 4000 seconds for a network with 605 TRX. The use of sampling reduced this time to about 50 seconds. As for the accuracy of the sampling approach, obviously increasing the sample size would improve the accuracy. A sample size of 100 points appeared to produce sufficiently accurate results for the networks tested.

For HSN-MAIO assignment the procedure we propose for the hop-synchronized network is similar in structure to the one used for the fully-synchronized network by incrementally building a partial solution. The main difference lies in the computation of the hopping collision rates as described above.

2.3. HSN-MAIO assignment in an asynchronous network

In an asynchronous network, and at any instance, the time difference between any TRX pair belonging to different cells can be anywhere within the sequence length. Thus, in order to compute the collision rates between sequences one can run the sequence generation procedure in GSM. In such a case a collision between two sequences would not necessarily occur for the full duration of a TDMA frame, but would only involve part of the frame. So the simulation would involve examining the system with a time line discretized to a very small level (a fraction of a time frame). Embedding such simulation into an optimization algorithm would be extremely time consuming.

For the investigation carried in this paper we determine the theoretical performance of the asynchronous network by computing expected collision rates based on the assumptions that the pseudo-random sequence generator used in GSM network procedures un-correlated sequences, and the frame numbers in different sequences are independent.

We denote by $f_i(t)$ the frequency used by TRX i at instant t. Based on the random sequence assumption, the probability that a frequency f is being active in TRX i is given by:

$$\text{Prob}\left(f_i(t)\right) = \begin{cases} 0, \text{if } f \notin MA_i \\ \dfrac{1}{|MA_i|}, \text{if } f \in MA_i \end{cases},$$

where MA_i is the set of frequencies on which TRX i is allowed to hop. Since sequence hop times are assumed independent, we have:

$$\text{Prob}\left(f_i(t) = f \text{ and } f_j(t) = g\right) = \begin{cases} 0, \text{if } f \notin MA_i \text{ or } g \notin MA_j \\ \dfrac{1}{|MA_i|}\dfrac{1}{|MA_j|}, \text{if } f \in MA_i \text{ and } g \in MA_j \end{cases}.$$

Under the assumptions of independence of the hop times and the randomness of the hopping sequences, one can then compute the joint probability distribution governing the hopping patterns of two sequences. This depends on the MAs only and is independent from any HSN-MAIO assignments. Using these computations, one can compute the expected co-channel and adjacent-channel collisions between two TRXs, and so we have:

- $C^{co}_{(is,jr)} = |\,\mathrm{MA}_i \cap \mathrm{MA}_j\,| \dfrac{1}{|\mathrm{MA}_i|} \dfrac{1}{|\mathrm{MA}_j|}$, $\forall s, r$ and
- $C^{ad}_{(is,jr)} = |\,\{f : \text{such that } f \in MA_i \text{ and } f \pm 1 \in MA_j\}\,| \dfrac{1}{|\mathrm{MA}_i|} \dfrac{1}{|\mathrm{MA}_j|}$, $\forall s, r$.

3. Experimental Results

In this section we examine the impact of optimizing resource allocation in the context of FH in a real-life network. The analysis is based on a network with 199 cells and a total of 605 TRXs (with a maximum of 4 TRXs per cell). The available radio bandwidth contains 40 frequencies. We are also provided with the MA allocation plan and the HSN-MAIO plan used by the operator

Table 1 summarizes the results of these experiments for the three levels of synchronization considered. This table compares a frequency hopping plan that was implemented by the operator to the optimized plans generated by the procedures proposed in this research (where a frequency hopping plan is composed of an MA allocation plan and an HSN-MAIO assignment plan). For each scenario, Table 1 provides the average, maximum and standard deviation of the interference among TRXs in the network. The MA plan provided (implemented by the operator) allocates six frequencies to each cell for hopping in addition to 1 frequency for the non-hopping TRX. The optimized MA plan was obtained by running the MA construction procedure (described in Bourjolly et al. (2002)) several times in order to obtain the best MA size. The best compromise between minimizing average interference and minimizing maximum interference was achieved with an MA of size six. For asynchronous networks, the procedure used to evaluate the performance of a given plan does not depend on the HSN-MAIO allocation, but depends on the MA allocation plan only. This explains why the evaluation results are the same for all HSN-MAIO plans with the same MA plan.

Table 1 compares several HSN-MAIO assignment plans: for the MA allocation provided by the operator, we compare a random HSN-MAIO plan, the actual plan used by the operator and optimized plans generated by the proposed heuristics. These are also compared with the use of optimized hopping sequences (for fully-synchronized network).

Note that the HSN-MAIO plan provided by the operator uses Cyclic Hopping for all cells. Computational times are reported for a Pentium III PC at 500MHz.

A first observation that can be made from Table 1 is that MA planning strongly determines the quality of the FH solutions. This holds for asynchronous, hop-synchronized or fully-synchronized networks. The improvement appears both as a reduction in the average as well as in the maximum interference. These experiments also indicate for all cases that introducing synchronization in the networks results in lower interference levels. We observe lower interference levels for hop-synchronized network than asynchronous network and further improvement for fully-synchronized networks. These gains may be due the fact the assignment algorithm is able to better exploit the control that is provided by increased synchronization. Comparing the optimized HSN-MAIO assignment plan to the random plan under hop-synchronized scenario, we see only a small gain. However, in a fully-synchronized network, optimizing HSN-MAIO assignments improves the solution quality. These observations indicate that because of the randomness in asynchronous or hop-synchronized networks, the resulting hopping sequences are about equally correlated. On the other hand, in fully-synchronized networks, the absence of randomness reveals the differences in correlation between hopping sequences, which can be exploited by the HSN-MAIO assignment procedure.

Inset Table 1 here

For fully-synchronized networks, we propose the use of "Optimized Hopping" (i.e., optimizing the hopping sequence as opposed to optimizing the HSN-MAIO assignments, see Bourjolly et al. (2002)). Results indicate that Optimized hopping provides a better approach than optimizing HSN-MAIO allocation for both MA allocation plans. The gain is more pronounced with optimized MA allocation.

This experimental analysis shows that optimization of the resource allocation under FH can significantly reduce the total interference. In addition, we observe that full-synchronization allows us to take further advantage of FH. The use of the sequence

generation procedure of GSM systems in the context of synchronized networks can be improved upon by actually building the hopping sequence through "Optimized Hopping".

4. Conclusion

We have examined the assignment problem that arises in the management of hopping sequences for frequency hopping GSM networks. The problem was formulated as an assignment problem and analyzed under 3 levels of network synchronization. For each of these scenarios, we have proposed heuristic procedures to evaluate and generate hopping plans. Experimental results indicate that the introduction of synchronization and careful frequency assignment are critical to reducing interference and taking full advantage from frequency hopping. In addition, in the case of a fully synchronized network, the proposed sequence management procedure significantly improves upon the solution used by the network operator for the data set at hand. Further analysis on more data sources are required to confirm these observations.

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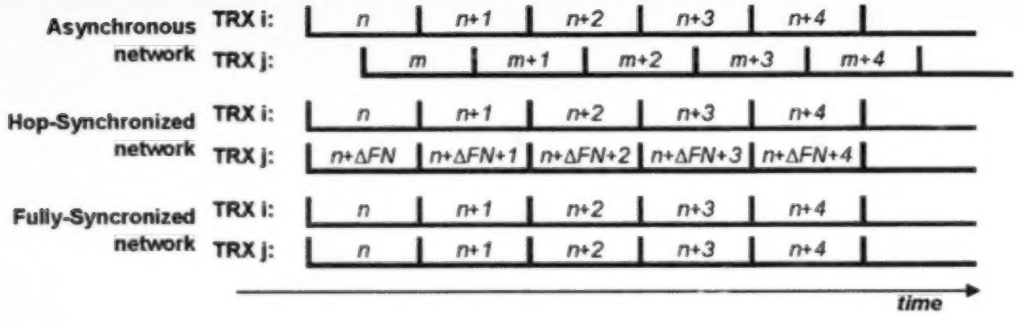

Figure1: FH: Network synchronization levels

		Asynchronous network	Hop-synchronized network	Fully- synchronized network
MA plan provided HSN-MAIO plan Provided	Avg. Max. StDev.	31.1 213.2 29.7 (<1 sec)	19.9 133.3 19.2 (304 secs)	19.3 147.6 21.4 (28 secs)
MA plan provided Random HSN-MAIO plan	Avg. Max. StDev.	31.1 213.2 29.7 (<1 sec)	19.9 129.0 19.1 (53 secs)	19.9 126.9 19.1 (28 secs)
MA plan provided Optimized HSN-MAIO plan	Avg. Max. StDev.	31.1 213.2 29.7 (<1 sec)	19.6 126.4 18.9 (39278 secs)	12.8 100.1 14.2 (7819 secs)
MA plan provided Optimized sequence	Avg. Max. StDev.	31.1 213.2 29.7 (<1 sec)	- - -	9.6 102.6 13.0 (28 secs)
Optimized MA plan Random HSN-MAIO plan	Avg. Max. StDev.	26.9 130.0 22.9 (<1secs)	15.4 43.8 11.6 (53 secs)	15.0 47.4 11.6 (29 secs)
Optimized MA plan Optimized HSN-MAIO plan	Avg. Max. StDev.	26.9 130.0 22.9 (<1secs)	15.1 42.9 11.3 (38770 secs)	9.0 40.0 8.1 (7496 secs)
Optimized MA plan Optimized sequence	Avg. Max. StDev.	26.9 130.0 22.9 (<1secs)	- - -	1.9 19.2 2.8 (29 secs)

Table 1: FH - HSN-MAIO management: Experimental results